Business and Economics

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Forecasting of Currency Exchange Rates Variance

The report reviews currency exchange rate forecast issues. For this reason, corresponding time series have been studied based on which features of this type of series have been determined. Taking into account nature of these features, several models have been processed for currency exchange rate forecasting. Comparing the results of the models, the best model is selected and used for estimate currency exchange rate's future movements.

Key words: forecasting, currency exchange rates; ARCH and GARCH models

Introduction

Trading currency positions can be considered to be a financial instrument. In financial trading, one of the key tasks is to try to capture the movement of the underlying asset, which is usually known as volatility. The volatility is the conditional standard deviation of the underlying assets return (r_t) and denoted by σ_t. This volatility depends on the trading each day and some previous days [1]. As with other financial time series, one of the main characteristics of the volatility of currency exchange return is that it appears in clusters (see Figure 1, 2 and 3). The second is that the volatility changes over time and in most cases stays within some spans. In other words, this kind of data suffers from heteroskedasticity.

In recent years, especially with regard to financial applications, ARCH [2] and Generalize ARCH (GARCH) models have received ample attention for dealing with heteroskedasticity [3]. The aim in this paper is to assess empirically the adequacy of this class of models in currency exchange return volatility forecasting. To accomplish this, we consider three currencies (USD, EUR and Georgian LARI) exchange rate sequences and evaluate how well the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model replicates the empirical nature of these sequences.

To assess the forecast accuracy of the GARCH model we need the time series to be stationary. One way to make financial time series stationary is to use continuously compound rate of return. If we denote the exchange rate at time t by P_t , we can transform the sequence of exchange rates as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1}),$$

where r_t - the continuously compound rate of return at time t. The compounded daily return, r_t can be computed simply by taking first difference of the natural logarithms of daily prices.

The GARCH (n, m) model can be expressed as:

$$\eta_t = \sigma_t \varepsilon_t ,$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{n} \alpha_i \eta_t^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 , \qquad (1)$$

where $\varepsilon_t \approx N(0,1)\ iid$, $\eta_t = r_t - \mu_t$, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter and

$$\omega \geq 0,\ \alpha_i \geq 0,\ \beta_j \geq 0\ and \sum_{i=1}^{\max(n,m)} (\alpha_i + \beta_i).$$

In this paper we use a large sample size (more than 2 300 observations) in order to get the best results when estimating standard errors even with heteroskedasticity. We will investigate if our large set of financial data can be fit to a time series model, and which model will provide the best fit. Figure 1, 2 and 3 show the continuously compounded daily returns, respectively, from XE: (1) (LARI / USD), (2) (USD / EUR), (3) (LARI / EUR). These figures show behavior of currency trading return and clearly demonstrate some kind of dependence between conditional variances in consecutive moments. In other words, there is an ARCH affect and we will examine GARCH model for these time series.

The GARCH model also takes into account volatility clustering and tail behavior, which are important characteristics of financial time series. It provides an accurate assessment of variances and covariances through its ability to model time-varying conditional variances. GARCH allows for modeling the serial dependence of the volatility. Due to the conditional property of GARCH, the mechanism depends on the observations of the immediate past, thus including past variances into explanation of future variances. Financial return volatility data is highly influenced by time dependence, which can cause volatility clustering. Time series such as this can be parameterized using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which can then be used to forecast volatility.

Figure 1 ln(LARI / USD)

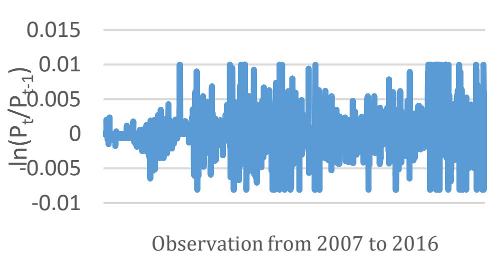


Figure 2 ln(USD / EUR)

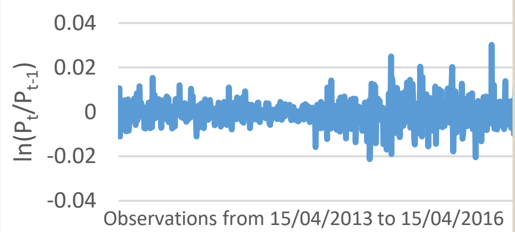
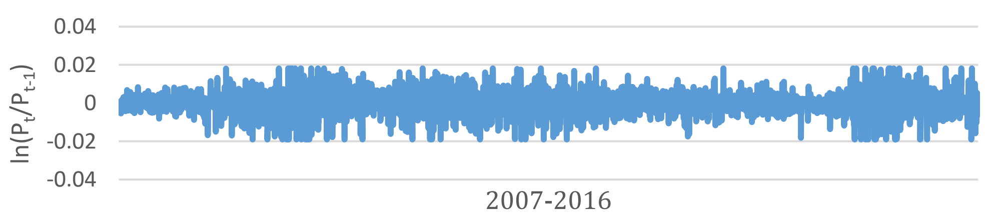


Figure 3 ln(LARI/ EUR)



All these three figures indicate that there are ARCH effects and there are some stationary parts and much more stationary parts. The financial return volatility data is highly influenced by time dependence, which, in these cases, evidenced in volatility clustering. We use GARCH class models for time series such as and parameterize it and forecast these three currencies volatility.

We could have easily performed a transformation on a non-stationary data set to make it stationary. This process is called differencing. The most basic method of differencing consists of simply taking the difference between consecutive observations.

USD/EUR Exchange Rate

We used GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2) models and have obtained following results:

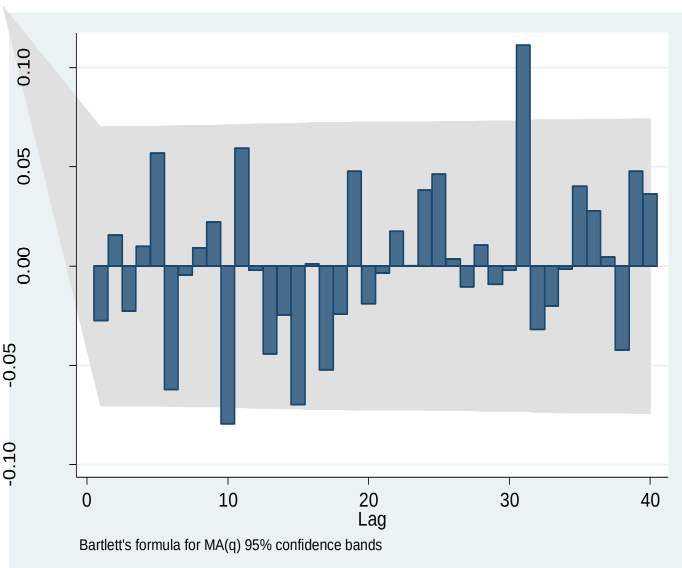
1. GARCH (1, 1): $\sigma_t^2 = 0.0011093 + 0.0354899\,\eta_{t-1}^2 + 0.9612536\,\sigma_{t-1}^2$

2. GARCH (2, 1):
$$\sigma_t^2 = 0.0010388 + 0.0868201\,\eta_{t-1}^2 - 0.0560561\,\eta_{t-2}^2 + 0.9660324\,\sigma_{t-1}^2$$

3. GARCH (1, 2):
$$\sigma_t^2 = 0.0020431 + 0.0628202\,\eta_{t-1}^2 + 0.1186592\,\sigma_{t-1}^2 + 0.8124835\,\sigma_{t-2}^2$$

4. GARCH(2,2):
$$\sigma_t^2 = 0.0012239 + 0.0894452\,\eta_{t-1}^2 - 0.0526774\,\eta_{t-2}^2 + 0.78835538\,\sigma_{t-1}^2 + 0.1711424\,\sigma_{t-2}^2$$

All these model have the same ACF functions as shown in next figure:



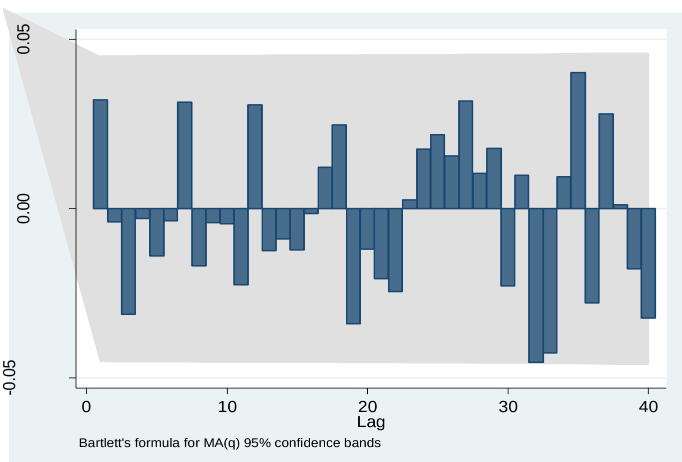
Except for two residuals, all others are within 2 standard deviations of the sample autocorrelation. GARCH (1, 2) and GARCH (2, 2) do not fit the conditions given in (1). For the left, now we have to check normality of these models residuals distribution. If we look at their residuals skewness and kurtosis, we will see that both are slightly skewed to the right side (0.03), but both have about the same kurtosis of about 2.77 which do not give enough arguments to reject formality of their residuals.

LARI/EUR Exchange Rate

Let consider the same GARCH models as previous.

1. GARCH (1, 1): $\sigma_t^2 = 0.0019377 + 0.0473765\,\eta_{t-1}^2 + 0.9486566\,\sigma_{t-1}^2$
2. GARCH (2, 1):
$$\sigma_t^2 = 0.0019363 + 0.0484219\,\eta_{t-1}^2 - 0.0011433\,\eta_{t-2}^2 + 0.9487533\,\sigma_{t-1}^2$$
3. GARCH (1, 2):
$$\sigma_t^2 = 0.0019851 + 0.0484657\,\eta_{t-1}^2 + 0.9234984\,\sigma_{t-1}^2 + 0.0239673\,\sigma_{t-2}^2$$
4. GARCH(2,2):
$$\sigma_t^2 = 0.0034587 + 0.0444576\,\eta_{t-1}^2 + 0.04026644\,\eta_{t-2}^2 + 0.16149546\,\sigma_{t-1}^2 + 0.7466865\,\sigma_{t-2}^2$$

Following figure of ACF Plots of the Residuals is:



For all these GARCH models residuals' skewness are almost the same with negative sign (-0.098) and kurtosis – 2.93. Hypothesize that residuals of these models follows a normal distribution we can't reject based on these evidences. In this case, only GARCH (1, 1) model is appropriate.

LARI/USD Exchange Rate

The ACF, as the name implies, shows a self (auto) correlation or relationship among the observations. The next Figure 4 gives evidence that shows the existence of autocorrelation in this time series. In other words, there is a serial dependence in the variance of the data. A geometrically decaying ACF plot would indicate that we should use some possibly a combination of an AR and MA model. Notice that the fist lag of the ACF plot is close to zero, indicating that our data set does not appear to have much correlation between observations. The PACF (see Figure 5) is used to determine the appropriate order of a fitted ARIMA data set. The PACF is used to determine the appropriate order of a fitted ARIMA data set. We can check this by looking at the plot of the partial autocorrelation function (PACF). The most we could expect from an ARIMA model would be MA (2) function.

Figure 4 The autocorrelation function

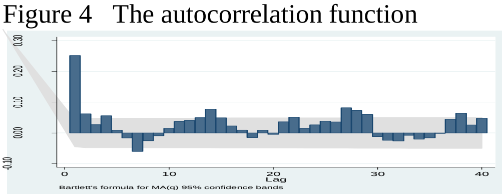
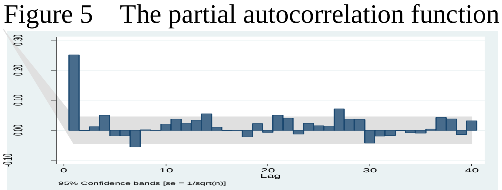


Figure 5 The partial autocorrelation function



The autocorrelation function of MA (2) model's residuals is shown in Figure 6. By viewing the ACF and PACF, the evidence is weak towards finding a good fitting AR model for the data. According to the ACF and PACF the data looks almost random (see Figure 7) and certainly shows no easily discernible patterns. This would support the appearance of the time series plot since the plot looks a lot like white noise except for the change in the spread (variation) of observations. Such heteroskedasticity would most likely not be evident in a truly random data set.

Figure 6 MA (2): Moving Average Models

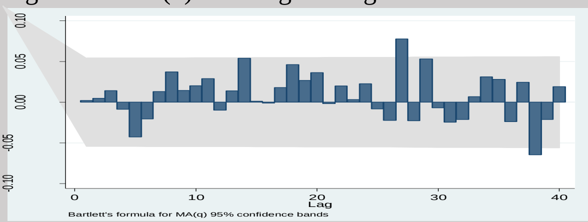
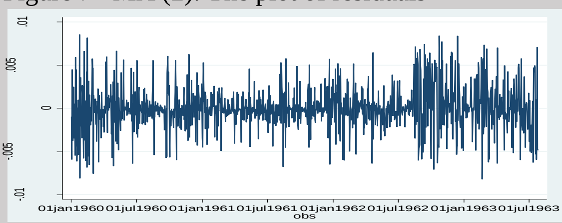


Figure 7 MA (2): The plot of residuals



Now we will combine GARCH model with MA (2). The results of this are:

MA (2) & GARCH (1, 1):

$$r_t = 0.01094 + 0.3245992\ \varepsilon_{t-1} + 0.1330251\ \varepsilon_{t-2}$$

$$\sigma_t^2 = 0.000146 + 0.1258626\ \eta_{t-1}^2 + 0.8883365\ \sigma_{t-1}^2$$

We don't represent other models because of their parameters some of which is insignificant and their less fitted characteristics to given time series patterns.

Our choices for the best models in above sections are based on assessing the residuals of the considered models. For this goal, we looked up the ACF plots of residuals, probability plots of the residuals and assessed each model with respect to the Ljung – Box statistic. Then, to check the normality assumption of the errors, we used the normal probability plots and histograms of the fitted GARCH models which showed that their errors are very close to normal distribution. The skewness and kurtosis values did not show exactly symmetric matters of errors but tails are not too much heavier than normal distribution. In addition, we simulated data from all GARCH models and evaluated the simulation data with respect to the given empirical time series. The comparison of these characteristics of considered models shows that in common, the GARCH (1, 1) model was the best in all cases.

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