Scenario Logic and Probabilistic Models of Bribes

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Abstract

True logic of our world is calculation of probabilities.
Maxwell, J. C.

The scenario logic and probabilistic (LP) bribe models for the department "Economic crimes" of towns are offered with the purpose of revealing, estimating and analyzing bribes on the basis of the statistical data. The following bribe LB-models are described: 1) at the institutions according to the results of their functioning, 2) of the officials on the basis of the descriptions of their behavior, 3) of the institution and of the officials on the basis of the analysis of the service parameters. Examples of identifying and of the analysis of the bribe LP-models according to the statistical data are given here. Problems of bribes and corruption are of great computing complexity and are solved only by means of special Software.

სცენარის ლოგიკა და ქრთამის ალბათობითი მოდელები

ე.დ. სოლოჟენცევი

სამყაროს ჭეშმარიტი ლოგიკა ალბათობების გამოთვლა არის.
დ.კ. მაქსველი

ეკონომიკური დანაშაულების საქალაქო დეპარტამენტისთვის სცენარის ლოგიკასა და ქრთამის ალბათობითი მოდელების წარმოდგენა მიზნად ისახავს ქრთამის აღების გამოაშკარავებას, შეფასებასა და ანალიზს სტა-ტისტიკურ მონაცემებზე დაყრდნობით. ქრთამის ლოგიკური და ალბათობითი LB მოდელის აღწერა წარმოდგენილია: 1) ორგანიზაციებში, მათი ფუნქციონირების შედეგების მიხედვით, 2) მაღალჩინოსნების მიერ მათი ქცევის აღწერისა და ანალიზის საფუძველზე, 3) ორგანიზაციების და მაღალჩინოსნების მომსახურების პარამეტრების გაანალიზების საფუძველზე. ნაშრომში მოყვანილია სტატისტიკურ მონაცემებზე დაყრდნობით, ქრთამის აღების ლოგიკურ-ალბათობითი მოდელების ამოცნობის და გაანალიზების მაგალითები. ქრთამის აღება და კორუფცია თავისი ხასიათით გამოირჩევა გამოთვლითი სირთულით. დღევანდელ სინამდვილეში ამ პრობლემის გადაჭრა ხდება სპეციალური კომპი-უტერული პროგრამების საშუალებით.

Problems of bribes and corruption have been actual at all times and in all countries. Now there are a lot of articles on www.vzyatka.ru about the flourishing of bribes and about the corruption.
Books and articles on corruption and on bribes (Albrecht, Wernz, Williams, Fraud, 1995, p. 396; Satarov, 2004, p. 368), on social statistics (Eliseeva, 2004, p. 656; Heckman & Leamer, 2002) have thorough substantial descriptions and analysis, as well as a great number of various examples, comments on the law and on the criminal codex, but they do not contain any mathematical models of bribes.

The adequate mathematical apparatus is needed for the solution of social and organizational problems, including problems of revealing various frauds, bribes and corruption. It must be based, , according to John von Neumann and Norbert Wiener (Solozhentsev, 2006, p. 560), on logic, on discrete mathematics and on combination theory.

Such adequate mathematical apparatus is being developed and it is called "Logic and probabilistic (LP) theory of risk with the groups of incompatible events" (Solozhentsev, 2006, p. 560; Solojentsev, 2004, p. 391). It has been tested for the estimation and for the analysis of credit risks, security portfolio risk, the risk of the loss of quality, the risk of non-success of the management of the company. The LP-models of risk have a high quality. For example, the credit LP-models of risk have shown the accuracy almost two times higher, the robustness almost seven times greater than other methods and also an absolute transparency in classifying than other methods.

In the present work an attempt has been made to use the LP-approach and the LP-calculus (Solozhentsev, 2006, p. 560; Solojentsev, 2004, p. 391) for the solution of the actual problem - for the estimation and analysis of the probability of bribes and corruption.

1. Axioms of the bribe theory

Corruption is regarded as the basic kind of the so called shadow economy. More often corruption implies the reception of bribes and illegal monetary incomes by state bureaucrats who extort them from citizens for the sake of personal enrichment. That is a brazen violation of public morals and of the norms of law.
For the construction of the system and the technique of the struggle against bribes and corruption the following axioms have been accepted (Solozhentsev, 2006, p. 560; Solojentsev, 2004, p. 391) :

- Under the pressure of circumstances everyone may swindle if valuables are not guarded well enough and if it is possible to conceal the trickery for some time and when the control over the validity of the decisions taken is insufficient.
- Without a quantitative estimation and without the analysis of the probability of bribes it is impossible to struggle against the swin dle, bribes and corruption.
- Each commercial bank or company is capable of a swindle or cor ruption if there is no transparency in their business and no control over their activities.
- Behind the non-transparency of the techniques of the estimation of credit risks and ratings of the banks and of the borrowers there may be bribes and swindles.
- Complexity of the organizational structure of an institution or company can be a sign of swindle and corruption.

Concepts of the probability of bribes and corruption are close to those of reliability and safety in engineering and they are also close to the notion of risk in economy, in business and in banks. Most frequently bribes take place when people receive licenses (in education, tourism, medicine, construction), sanctions (GAI, customs), in education (certificates, diplomas, examinations), registration (bodies of the Ministry of Internal Affairs, embassy, bodies of local authorities), and etc.

The scenarios and the technique of a bribe are various for the ministry, for the mayoralty, for institutions, for companies, for banks, for officials, for doctors, for teachers, etc. The Bribe implies two objects: the briber and the bribe-taker, either of whom has his benefit. The briber solves his problem faster, more qualitatively, receives privileges, bypasses the law, etc. The bribe-taker has monetary or material benefit, etc. We use the following terms: probability of corruption and of a bribe, probability of success and of non-success, probability of the absence or of the presence of a bribe, probability of a good or bad project (of an object, of an official, of an institution) We consider those terms from the point of view of the size of the probability of a bribe.

For a quantitative estimation and for the analysis of the bribe probability we use the logical and probabilistic non-success risk LP-theory (LP-theory) with groups of incompatible events (GIE) (Solozhentsev, 2006, p. 560; Solojentsev, 2004, p. 391; Solojentsev, Karassev, Solojentsev, 1999, p. 120), and some bribe LP-models are constructed on the basis of the statistical data. The paper is one of the first mathematical publications on the probability of bribes and does not claim to consider all the aspects of this complex problem or to develop all the scenarios of bribes. Here we just give the description and the construction of the bribe model, try to give the estimation and the analysis of the probability of a bribe, and hardly ever touch upon the social, legal and organizational problems of bribes.

2. The LP-theory of a bribe with groups of incompatible events

Events and probabilities. An event of a bribe is described by signs and their grades, which happen to be random variables and are regarded as the logic variable of casual sign-events and grade-events having certain probabilities. The sign-events are connected by the logic connections OR, AND, NOT and can have cycles. Grade-events for a sign make a group of incompatible events (GIE) (Solozhentsev E., 2006, p. 560; Solojentsev E., 2004, p. 391).

Signs are the characteristics of an object (of a process, of a project) for which special measurement scale-grades are used: the logical scale (the truth\the lies, 1\0), the qualitative scale (high, average and low salary), the numerical scale (intervals [a, b], [b, c]), etc. Generally, the grades are linearly disordered and it is impossible to tell whether grade 3 is worse or better for the final event than grade 4.

The logical variable Zj corresponding to a sign-event, is equal to 0 with the probability Pj if the sign j testifies that a bribe has taken place there, and it is equal to 1 with the probability $Qj=1-Pj$ in case of the absence of a bribe. The logical variable Zjr corresponding to the grade r for the sign j, is equal to 0 with the probability Pjr and is equal to 1 with the probability $Qjr=1-Pjr$. The vector $Z(i)=(Z1,\ldots,Zj,\ldots,Zn)$ describes the object i on the basis of statistics. When the object i is given instead of the logical variables $Z1,\ldots,Zj,\ldots,Zn$, it is necessary to substitute the logical variables Zjr for the grade of the sign of this object.

The L-function of a bribe in a general way is as follows

(1)

$$Y=Y(Z_1, Z_2,\ldots,Z_n).$$

The P-function of a bribe in a general way is

(2)

$$Pi(Y=1|Z(i))=P(P_1,...,P_j,...,P_n),\ i = 1,2,...,N$$

Three probabilities have been considered for every grade-events in GIE (Fig. 1): P2jr is a relative frequency in the statistics; P1jr is a probability in GIE; Pjr is the probability, substituted into (2) instead of the probability Pj. We will determine these probabilities for the j-th GIE:

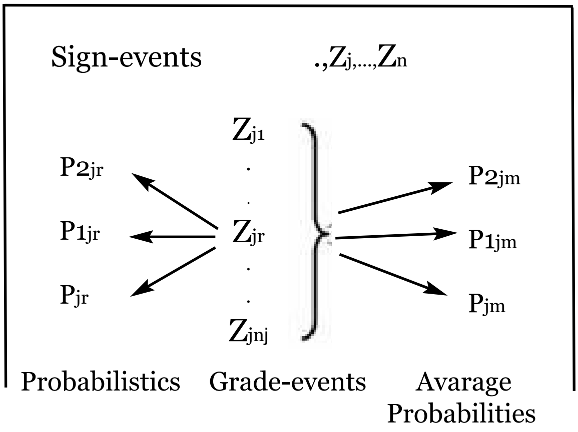


Fig.1. Probabilities in the groups of incompatible events

(3)

$$P2_{jr} = P\{Z_{jr} = 1\};\ \sum_{r=1}^{Nj} P2_{jr} = 1; r = 1, 2, ..., Nj;$$

(4)

$$P1_{jr} = P\{Z_{jr} = 1 \mid Z_j = 1\};\ \sum_{r=1}^{Nj} P1_{jr} = 1; r = 1, 2, ...Nj;$$

(5)

$$P_{jr} = P\{Z_j = 1 \mid Z_{jr} = 1\}; r = 1, 2, ..., Nj,\ j = 1, 2, ...n.,$$

where n is the number of signs; Nj is the number of grades in the j -

sign, the vertical hyphen (|) is read provided.
The average values of the probabilities P2jr, P1jr and Pjr for the gradation in GIE are equal to:

(6)

$$P2_{jm} = 1 / N_j ; P_{jm} = \sum_{r=1}^{Nj} P_{jr} P2_{jr}; \; P1_{jm} = \sum_{r=1}^{Nj} P1_{jr} P2_{jr}.$$

We will estimate the probabilities Pjr during the algorithmic iterative identification of the P-model of the bribe on the basis of the statistical data. In the beginning it is necessary to determine the probabilities P1jr satisfying (4) and to pass from the probabilities P1jr to the probabilities Pjr. The number of the independent probabilities is equal to:

(7)

$$N_{ind} = \sum_{j=1}^{n} N_j - n.$$

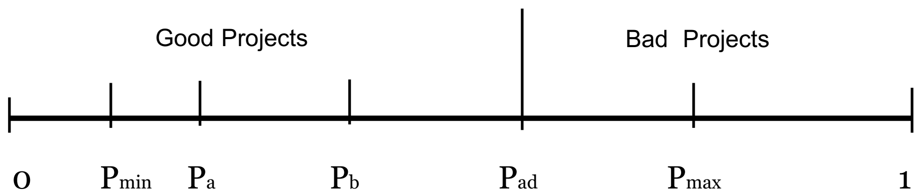
The probabilities Pjr and P1jr are connected with the Bayes formula for the case of a limited quantity of information by way of the average probabilities Pjm and P1jm [5]:

(8)

$$Pjr = P1_{jr} * (Pjm / P1_{jm}); r = 1, 2, ..., Nj; j = 1, 2, ..., n .$$

Identifying the bribe LP-model on the basis of statistical data. The problem of the identification of the bribes LP-model is solved by the algorithmic methods [5, 7]. The following scheme of solving the problem is proposed here. Let the probabilities for the grades Pjr, r=1,2,...,Nj ; j=1,2,...,n be known as a first approximation; and the risks Pi, i=1,...,N for the objects in statistics be also calculated, each of which might be accompanied by bribes. In the statistics of good projects the symbol Ng is used and in the statistics of bad projects the symbol Nb is used. We will determine the admitted risk Pad (Fig. 2) so that the number of projects accepted by us without bribes (the good projects) Ngc had the risk lesser than the admitted one and, accordingly, the number of the projects with bribes (the bad projects) Nbc=N - Ngc had the risk which exceeded the admitted one. At the step of optimization we shall change the probabilities Pjr, r=1,2,...,Nj; j=1,2, ...,n in such a way that the number of the recognizable projects might be increased. The variables Pad and Ngc are connected unequivocally. In the algorithm of the problem it is more convenient to use Ngc and to determine the admitted risk Pad.

Fig. 2. The scheme of the classification of projects



The condition Pi>Pad specifies the following types of projects: Ngg - denotes the projects which are good according to both - their methods and the statistics; Ngb - denotes the projects which are good according to their methods and bad by the statistics; Nbg - denoted the projects which are bad by their methods and good by the statistics; Nbb - denotes the projects bad by both the methods and the statistics. The risks of the projects Ngg, Nbb, Ngb, Nbg move relative to Pad at the change of Pjr. At the transition of same projects which are bad to the right from Pad on value of the risk, the some number of projects passes to the left. The change Pjr which translates the projects Ngb and Nbg through Pad towards each other will be the optimal one.

The problem of the identification of the P-model of the bribe is formulated like this (Solozhentsev E., 2006, p. 560; Solojentsev E., 2004, p. 391).

The statistics on the bribes, having Ng good and Nb bad projects and the bribe P-model (2) have been assigned. It is required to determine the probabilities Pjr, r=1,2,...,Nj; j=1,2,...,n grade-events and the admitted risk Pad , dividing the projects into the good and bad ones. The goal function is as follows: the number of the projects which are to be correctly classified should be maximal

(9) $$F = N_{gg} + N_{bb} \rightarrow \underset{Pjr}{MAX},$$

From expression (9) it follows that the accuracy of the P-model of the bribe in the classification of the good objects Eg , the bad

objects Eb and as a whole Em is equal to
(10)

$$E_g = N_{gb} / N_g; E_b = N_{bg} / N_b; E_m = (N - F) / N.$$

Restrictions:
1) The probabilities Pjr should satisfy the condition:

(11)

$$0 < P_{jr} < 1, j = 1, 2, ..., n; r = 1, 2, ..., Nj.$$

2) The average risks of the projects on the P-model and according to statistics should be equal; at identifying the P-model of risk we shall correct the probabilities Pjr on a step by the formula

(12)

$$P_{jr} = P_{jr} * (P_{av} / P_m); j = 1, 2, ..., n; r = 1, 2, ..., Nj,$$

where: Pav=Nb / N is the average risk according to statistics; Pm is the average risk on the model.
3) The admitted risk Pad should be determined at the given factor of asymmetry of the recognition of the good and bad projects, equal to

(13)

$$E_{gb} = N_{gb} / N_{bg}.$$

The formula for identifying the bribe risk LP-model

(14)

$$\Delta P1_{jr} = K_1 * \frac{N_{opt} - N_v}{N_{opt}} * K_3 * P1_{jr}, j = 1, 2, ..., n; r = 1, 2, ..., N_j,$$

where: K1 is the factor equal approximately to 0,05; Nopt, Nv are the number of optimization and the number of the current optimization respectively, K3 is a random number in the interval [-1, +1]. During the optimization the size P1jr tends to zero.

The analysis of bribe probability. Let the bribe P-model and the probabilities Pjr be defined and known. We shall determine the contributions of sign-events and grade-events into the probability of the bribe for the project and for a set of projects, and also into the accuracy of the bribe LP-model. For this purpose we shall calculate the differences between the values of the characteristics for the optimal model and if the probabilities are endowed with the corresponding grade-events of zero values (Solozhentsev E., 2006, p. 560; Solojentsev E., 2004, p. 391).

The contribution of a sign (of all the grades of a sign) into the probability of a bribe for the project *i*

(15)

$$\Delta P_j = P(i) - P(i)\Big|_{P_j=0}, \; j = 1, 2, \ldots, n.$$

The contribution of an attribute into the average probability of the bribe Pm of a set of projects

(16)

$$\Delta P_{jm} = P_{jm} - P_{jm}\Big|_{P_j=0}, \quad j = 1, 2, \ldots, n.$$

The contribution of an attribute into the goal function Fmax

(17)

$$\Delta F_j = F_{\max} - F\Big|_{P_j=0}, \quad j = 1, 2, \ldots, n.$$

Contributions of grades into the goal function Fjr will be calculated by us by analogy with (10) as mistakes of the classification of projects on every grad-event:

(18)

$$E_{jrg} = N_{jrgb} / N_{jrg}; E_{jrb} = N_{jrbg} / N_{jrb};$$
$$E_{jrm} = (N_{jrgb} + N_{jrbg}) / N_{jr},$$

where Njrgb, Njrbg, Njr are the numbers of the non-correct good and bad project and of all the projects with the grade r of the sign j.

By the results of the analysis of the contributions into the probability of the bribe grades, of the signs, of the project or a set of projects, the bribe model for the increase of its accuracy is optimized.

3. The bribe LP-model at an institution

The institution making decision on some projects (on the cases or affairs of the citizens). There are a lot of projects. The projects are either successful (good) or non-successful (bad). The reasons for the non-success of the projects are the unjustified sanctions, given out as a result of bribes.

Elements of the scenario and of the bribe LP-model are the functional departments Z1,...,Zj,....,Zn, each of which has Nj officials who make decisions.

Generally the object with the elements Z1,...,Zj,....,Zn is complex as it includes connections OR, AND, NOT, and repeated elements and cycles. Officials in the j-department Zj1,...,Zjr,...,ZjNj are GIE. The official, making a decision, signs the corresponding document. The construction of the bribe LP-model consists in the calculation of the probabilities Pjr, j=1,2,...,n; r=1,2,..., Nj with which officials take bribes on the basis of the statistics from N successful and non-successful projects.

We shall consider the bribe LP-model, say, of a bank. The statistics about the success of the credits is used. The reasons of the non-success of the credits are explained by bribes.

Let the bank has five functional groups of the officials who take decisions on giving out the credits. The logic variables Z1, Z2, Z3, Z4, Z5 correspond to these functional groups. These groups have accordingly N1, N2, N3, N4, N5 officials taking decisions. The number of the officials in groups coincides with the number of the grades in GIE.

The given credits are either successful (grade 1) or non-successful (grade 0). There are documents on the given credits where the officials making decision fix their signatures.
The greatest number of any possible combinations of the client's passing through the institution and the bribes is equal to

(19)

$$N_{max}=N_1+N_2+N_3+N_4+N_5.$$

The logic function of the bribes in the perfect disjunctive normal form (PDNF) has Nmax of the logical terms and we may write

(20)

$$Y = Z_1Z_2Z_3Z_4Z_5 \vee \overline{Z}_1Z_2Z_3Z_4Z_5 \vee Z_1\overline{Z}_2Z_3Z_4Z_5 \vee \cdots \vee \overline{Z}_1\overline{Z}_2\overline{Z}_3\overline{Z}_4\overline{Z}_5 .$$

Every logic variable from Z1, Z2, Z3, Z4, Z5 or its denial (the line over a variable) goes into any conjunct. All the conjuncts are orthogonal in pairs, that is PDNF is the orthogonal form of the logic function. At calculation of the probability of the event Y we put in (20) the probabilities P1, P2, P3, P4, P5 instead of the events Z1, Z2, Z3, Z4, Z5 and the sign "OR" replace by "+".

PDNF is the cumbrous recording the logic function. In really the logic bribe model may be recorded simpler if taking into account of the structure of bank departments and their connection. It can be of any kind. To be specific, we will assume that the structure of the risk model is presented by the "bridge"(Fig. 3).

The officials from Z1 and Z2 check the maintenance of the credits, and the officials from Z3 and Z4 take the decision on the size and on the terms of the credit. The top officials (chiefs) from Z5 control the process. The client visits one of the top officials who either advises the client or takes a bribe and directs the client to the officials from the groups Z3 or Z4 who take bribes too.

The logic model (L-model) of bribes in disjunctive normal form (DNF) (the records of the logic functions without the brackets) on the basis of the shortest way of functioning is

(21)

$$Y = Z_1Z_3 \vee Z_2Z_4 \vee Z_1Z_5Z_4 \vee Z_2Z_5Z_3.$$

The probabilistic model (P-model, P-polynomial) of bribes, obtained after the orthogonalisation of the logic function (19)

(22)

$$P = p2p4 + p1p3 + q1p2p3q4p5 + p1q2q3p4p5 - p1p2p3p4.$$

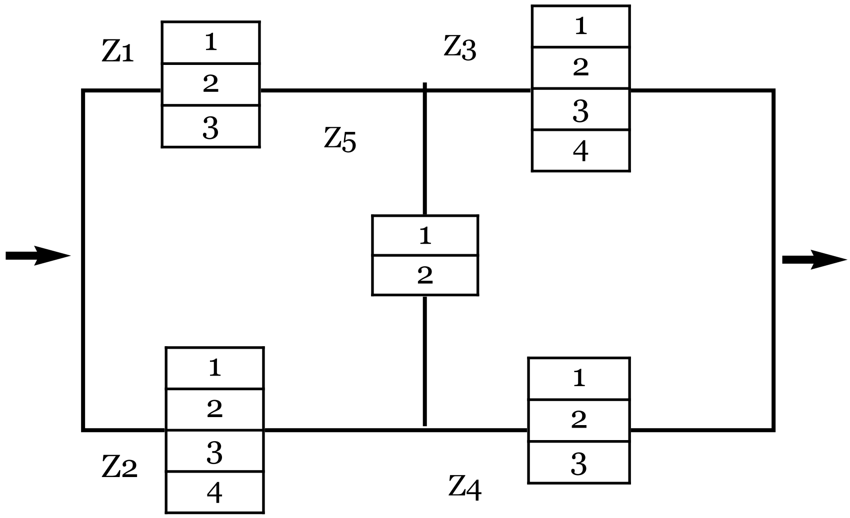


Fig. 3. Structural model of the bribes of the “bridge” type

Example 1. For training the bribe P-model the statistics from 1000 credits is used (700 are good and 300 are bad). The average bribe risk is equal to Pav=300/1000=0,3. Five sign-events have from 4 to 11 grades; all in all there are 40 grades.

As a result of training, the probabilities Pjr and P1jr for all grade-events have been obtained and the following parameters of the bribe LP-model have been calculated: the goal function is equal to Fmax=720 and the admitted risk is equal to Pad=0,3094.

Some results of computing researches are given in Tables 1 and 2. The probabilities P2jr and P1jr of the grades though they make a total 1 in GIE, can differ essentially (Table 2). The probabilities of bribes (probabilities Pjr) differ more than 10 times. The sign-events 1 and 4 have the maximum average probabilities Pjm. The same events bring the maximum contributions to the average risk Pm. The average probabilities Pjm for sign-events differ nearly two times.

Table 1.
Average probabilities of bribes for the groups of officials

Groups, j	Probabilities, Pjm	Probabilities, P1jm	Number of officials, Nj
1	0.478113	0.249540	4
2	0.348310	0.075949	10
3	0.298833	0.133823	5
4	0.388857	0.116348	11
5	0.291868	0.091775	10

Table 2.
Probabilities of bribes of officials

Numbers of grades	Probabilities, P1r	Probabilities, P11r	Frequencies, P21r
Group Z1			
1	1.0	0.522300	0.274
2	0.596084	0.311103	0.269
3	0.248278	0.129579	0.063
4	0.070927	0.037017	0,394
Group Z2			
1	0.0	0.0 0,0	
2	0.687703	0.149933	0.014
3	0.227359	0.0495688	0.002
4	1.0	0.218209	0.054
5	0.510577	0.111316	0.017
6	0.704722	0.153643	0.086
7	0.570149	0.124304	0.057
8	0.448856	0.097859	0.224
9	0.434821	0.094799	0.187
10	0.001675	0.000365	0.359

4. The bribe LP-model on the basis of the description of the officials' behavior

A bribe is not a crime which is made a parade. There is no question about “corpus delict” at a robbery of a bank which is witnessed by the employees or by the clients. A bribe differs from any other kinds of crime by the difficulty of its revealing. However, bribes have a mass character and there are many data on bribes both in judicial law-courts, and in the controlling units.

For each type of bribes it is possible to find signs (Albrecht, Wernz, Williams, Fraud, 1995, p. 396; Solozhentsev E., 2006, p. 560) which are associated with a similar crime. Each of such signs has at least 2 grades. The P-model bribe can be identified on the statistical data.

The investigation of the bribe can be carried out only in the case when there are serious reasons to believe that the bribe had actually taken place in the past. The value of this “seriousness” can be estimated quantitatively on the probability of the bribe, and the final decision is taken by the head of the office.

Special signs testify to the bribes taken by the officials (doctors, teachers). There are the following signs of the person's unusual behavior:

- Age;
- Duration of the period of work at an institution or in a company;
- Purchase of a house, of an apartment, of a summer residence, of a car, etc. at the price not adequate to the level of the wages;
- Debts;
- Financial inquiries;
- Predisposition to gambling;
- The way of life beyond the habitual frameworks;
- Unusual behavior;
- Presence of complaints;
- Vague or criminal past;
- Dishonest or unethical behavior at the office;
- Absence of the division of duties;
- Absence of independent checks;
- Absence of the proper authority;
- Absence of the necessary documents and records;
- The neglect of the existing rules;
- The inadequate system of document circulation, etc.

Elements of the scenario and of the bribe LP-model, listed above, are presented by the signs Z1,...,Zj,...,Zn,, each of which has several grades. The grades for the j-sign of Zj1,...,Zjr,...,ZjNj form GIE. The bribe scenario of the official is described as follows: a bribe can take place if any sign-event takes place or if any two sign-events or all sign-events take place. The scenario of a bribe is given in Fig.4 in the form of a structural graph.

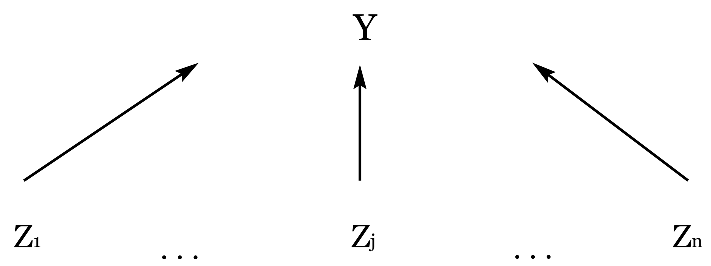


Fig.4. Structural model of bribe (-the logical circuit OR)

The construction of the bribe LP-model consists in the calculation of the probabilities Pjr, j=1,2,...,n; r=1,2,...,Nj (with which the official takes bribes) on the statistics of the bribe facts established by the courts.

The bribe L-function (the bribe LP-model) in DNF is (Solozhentsev E., 2006, p. 560; Solojentsev E., 2004, p. 391)

(23)

$$Y = Z_1 \vee Z_2 \vee \ldots \vee Z_j \vee \ldots \vee Z_n$$

The bribe L-function in the equivalent orthogonal form (ODNF) after the orthogonalization (23) is

(24)

$$Y = Z_1 \vee Z_2 Z_1 \vee Z_3 Z_2 Z_1 \vee$$

The bribe P-function (model, polynomial) is

(25)

$$P= p1+p2q1+p3q1q2+\ldots$$

“Arithmetics” in the bribe P-model is such that for the final event the bribe probability value is within the limits of [0,1] at any values of the probabilities of initiating events. For every grade-event in GIE (Fig. 1).we use the three probabilities P2jr, P1jr, Pjr , introduced before.

The maximum number of different bribes is equal to

(26)

$$N_{max} = N_1 + N_2 + .. + N_j + ... + N_n,$$

where N1,...,Nj,...,Nn are the numbers of the grades in the signs. If the number of signs is equal to n=20 and each sign has Nj=5 grades, the number of different bribes (conjuncts in a perfect disjunctive normal form - PDNF) equals the astronomical number Nmax=520, that explains the difficulties of the struggle with bribes and corruption. (PDNF comprises various conjuncts, each of which comprises all the variables Z1, Z2,..., Zn or their denials. The conjuncts are connected by the logical operation OR).

The bribe LP-model (23-25) describes all kinds of bribes and is the most complete and accurate one. In some cases, however, it is not necessary to take into account all possible bribes. For example, it is known from the statistic data that there ware bribes when one or two events occurred from Z1, Z2,...,Zn. Then, to simplify the model, you should use the bribe model for a limited number of bribes.

If we have a logical bribe model of four elements

(27)

$$Y = Z_1 \vee Z_2 \vee Z_3 \vee Z_4,$$

then for a limited number of bribes, when either one or two events occur, the bribe model will be recorded as

(28)

$$Y = Z_1 \overline{Z_2}\,\overline{Z_3}\,\overline{Z_4} \vee Z_2 \overline{Z_1}\,\overline{Z_3}\,\overline{Z_4} \vee Z_3 \overline{Z_1}\,\overline{Z_2}\,\overline{Z_4} \vee Z_4 \overline{Z_1}\,\overline{Z_2}\,\overline{Z_3} \vee Z_1 Z_2 \overline{Z_3}\,\overline{Z_4}$$
$$\vee Z_1 Z_3 \overline{Z_2}\,\overline{Z_4} \vee Z_1 Z_4 \overline{Z_2}\,\overline{Z_3} \vee Z_2 Z_3 \overline{Z_1}\,\overline{Z_4} \vee Z_2 Z_4 \overline{Z_1}\,\overline{Z_3} \vee Z_3 Z_4 \overline{Z_1}\,\overline{Z_2}$$

In the bribe L-model, all the logical summands are orthogonal in pairs, which allows the bribe P-model (P-polynomial) to be written directly:

(29)

$$p\{Y\} = p1q2q3q4 + p2q1q3q4 + p3q1q2q4 + p4q1q2q3 + p1p2q3q4 +$$
$$+ p1p3q2q4 + p1p4q2q3 + p2p3q1q4 + p2p4q1q3 + p3p4q1q2.$$

Example 2. The author did not have any factual data about the bribes, established by the courts on criminal cases. The modeling data were used as the statistical data. From 1000 officials, suspected of bribes, against whom suits were brought, only 300 were condemned, and 700 were considered to be innocent. Thus, the average risk of bribes is equal to Pav=300/1000=0,3. The suspected officials are described by n=20 signs whose total sum is 94 grades.

The identification of the bribe Â-model (25) consists in defining the probabilities Pjr, r=1,2,…,Nj; j=1,2,…,n for grade-events. The bribe probability for every suspected official is calculated on the optimization step and is compared to the allowable risk Pad. The suspected official is either bad or good. The goal function is formulated as follows: the number of the correctly classified suspected officials should be as great as possible.

Contributions of the grade-events into the accuracy of the bribe LP-model will be considered by us by the example of sign-events (Table 3) of the signs Z2 and Z13 for the optimal identified bribe LP-model (Fmax=826). The grade frequencies for all P2jr, for the bad P20jr and for the good P21jr , the probabilities of the grad-events P1jr and Pjr; for the mistakes of recognition on grades for all Ejr, for the bad E0jr and for the good E1jr officials who are under suspicion, are summerized in Table 3;

Table 3.
Probabilities and errors of recognition for grade-events of suspected officials

P2jr	P20jr	P21jr	P1jr	Pjr	Ejr	E1jr	E0je
Sign Z2							
0.014	0.007	0.007	0.010	0.019	0.214	0.429	0.0
0.002	0.001	0.001	0.070	0.014	0.500	1.0	0.0
0.054	0.032	0.022	0.194	0.038	0.278	0.682	0.0
0.017	0.005	0.012	0.159	0.031	0.412	0.5	0.2
0.086	0.038	0.048	0.145	0.028	0.256	0.417	0.053
0.057	0.019	0.038	0.095	0.019	0.228	0.289	0.105
0.224	0.066	0.158	0.067	0.013	0.169	0.196	0.106
0.167	0.056	0.131	0.053	0.010	0.203	0.183	0.250
0.359	0.076	0.283	0.016	0.003	0.114	0.081	0.237
Sign Z13							
0.0190	0.080	0.110	0.283	0.027	0.237	0.345	0.087
0.511	0.142	0.369	0.233	0.021	0.186	0.201	0.148
0.248	0.065	0.183	0.093	0.008	0.113	0.082	0.200
0.028	0.007	0.021	0.346	0.032	0.178	0.238	0.0
0.023	0.006	0.017	0.044	0.004	0.217	0.117	0.5

The contribution of the sign-event into the probability of a bribe by an official is proportional to the probability Pj, j=1,2,…,n, which equals the probability of the grade-event Pjr. The probabilities Pjr of sign grades differ more than 10 times. The grade errors Ejr in the classification of bribes differ almost 5 times.

The LP-analysis of the bribe model is carried out with the use of (15-18). For each sign j (Table 4): the average values of probabilities P1jm and Pjm, were determined and also the decrease of the number of the identified good and bad suspected officials Fj . When this sign was excluded from the bribe model, the bribe LP-model was retrained. The decrease of the number of the suspected officials that could be recognized is determined in relation to the optimal trained bribe model with all signs.

Table 4.
The contributions of the signs into the accuracy of the bribe model

Signs, j	Number of grades,Nj	P1jm	Pjm	Fj
1	4	0.272384	0.020226	-64
2	10	0.063346	0.012359	-27
3	5	0.098475	0.009327	-18
4	11	0.090820	0.020927	-26
5	10	0.080377	0.017593	-20
7	5	0.272148	0.022466	-20
7	5	0.206945	0.018549	-6
8	4	0.266619	0.017736	-6
9	4	0.183897	0.014253	-10
10	3	0.318015	0.018295	-10
11	4	0.251871	0.018974	0
12	4	0.247375	0.017166	0
13	5	0.206718	0.018900	-16
14	3	0.235637	0.014733	-2
15	3	0.261648	0.017591	-8
16	4	0.341959	0.021975	-2
17	4	0.289853	0.018739	0
18	2	0.482499	0.017417	0
19	2	0.508613	0.018138	0
20	2	0.750896	0.018326	-2

The maximum contribution into the accuracy of the suspected officials is brought with the sign-events: Z1, Z2, Z4, Z5, Z6, Z3, Z13. The zero contribution is brought by the sign-events Z11, Z12, Z17, Z18, Z19; excluding sign-events 11,12,17 and 18 reduces the number of the identified suspected only by 4.

The accuracy of the bribe LP-model changes with the change of the number of grades in a sign. The sign Z2 which in the initial variant had 10 grades was investigated. After retraining the bribe model, the following results were obtained: in the absence of the sign Fmax=800; with two grades Fmax=808; with four grades Fmax=812; with ten grades Fmax=824; with hundred grades (in that case there were seventy empty grades) Fmax=828.

We built a graph of bribe probabilities for 1000 suspected officials before and after sorting according to the value of the probability. Approximately 15 % of the suspected officials had small bribe probabilities and are good, and 15 % of suspected officials had high bribe probabilities and are very bad. It shows that it is necessary to classify the suspected officials according to the bribe probability value into four classes.

5. The bribe LP-model on the basis of the analysis of service parameters

Let's estimate the bribe probability using the statistics of the service parameters. These parameters can be, for example, the time it takes the official to solve the problem or it takes the dentist to make a denture (from the very beginning till the end of the process). Such statistics should contain the service precedent number, sufficed for the construction of the discrete or analytical distribution function.

Let us have the statistics (service times) for N clients Yi, i=1,2,...,N. If we construct a normal distribution law for the parameter Yi with the average value and the dispersion, it will lead to an essential decrease of the bribe estimation accuracy.

The service parameter can have either a continuous or discrete values. In both cases, with the purpose of increasing the bribe model adequacy and using the apparatus of LP-calculation, we shall build the discrete distribution on the chosen intervals of splitting the parameter values. We give the grade number for each interval. The grades make group of incompatible events (GIE). The probabilities of grade-events are determined by the formula

(30)

$$P_j = N_j/N,$$

where: Nj is the number of the parameter values in the statistics with the given grades; N is the number of the parameter values in the statistics.

The service parameter has the average value Ym and the allowable value Yad (Fig. 5).The probability P {Y <Yad} will be named by us the bribe risk. The scenario of the bribe is formulated as follows: if the service parameter is greater (smaller) than the allowable value, then we suspect that there must have been a bribe.

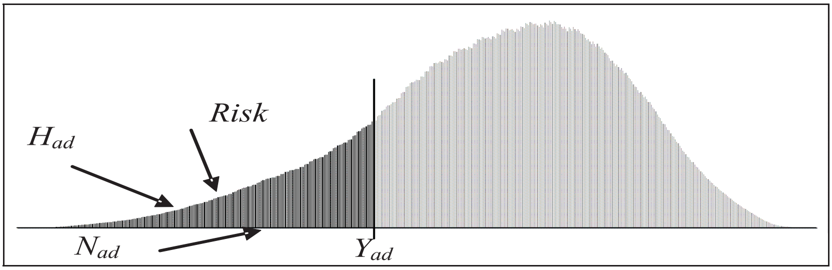


Fig. 5. Discrete distribution of the service parameter

Thus, for the service parameter at the given Risk we can compute the following: the admitted value Yad , the number of the values of the parameter in the “tail” of the distribution Nad, the entropy of the probabilities of the parameter in “tail” of the distribution Had.

In numerous publications in the field of VaR (Value-at-Risk) theory, the authors investigated tails of distribution. For these purpose various distributions and conditional probabilities are suggested, which have no rigorous justification. In the bribe LP-theory it is not to be done because we use any distribution laws given by a discrete line.

Example 3. N=700 clients have been served. The parameter Y1 determines the duration of the service by days and has N1=30 days-grades. The probabilities P1r are calculated, r=1,2,...,30 by (28). The admitted parameter value is Y1ad=10 and the risk value is Risk1=0,2. The suspicion of the bribe is caused if Y1<Y1ad.

Let there be one more the service parameter Y2 having N2=20 grades, the admitted parameter value Y2ad and the risk Risk2. The logical variables correspond to the service parameters. The logical variables can be dependent, but is not initially, for they are contained in the certain logic formula which determines the dependence between them.

For the case of two service parameters Y1 and Y2, we have N=N1*N2=30*20=600 combinations of service. The L-functions for two different service combinations Y1Y2 are orthogonal (the product of the logic functions of different combinations is equal to zero) as these combinations contain different grades for Y1 and Y2, belonging to GIE.

The property of orthogonality of different service combinations allows us to pass from the L-functions to the algebraic expressions for probabilities, that is the L-variables are to be replaced by the probabilities and the signs “or” are to be replaced by “plus”.
It is easy to calculate the number of combinations satisfying the bribe condition

(31)

$$P\{(Y_1 \le Y_{1ad}) \vee (Y_2 \le Y_{2ad})\},$$

and calculate the bribe probability for this condition.

Conclusion

Problems of identifying the bribe LP-models, the estimation and the analysis of bribe probabilities on the basis of the risk LP-theory with groups of incompatible events, as follows from expression (19, 26), have an extremely high computing complexity and can be solved only by means of modern computers and special Software. A complex of Software for the solution of all those problems of training, modeling and of the analysis of bribes has been elaborated. You may get the detailed information on these Software in (Solozhentsev E., 2006, p. 560; Solojentsev E., 2004, p. 391; Solojentsev E., 1999, p. 120; Karassev, V., Solojentsev V., 1999, p. 120), on www.inorisklab.com, www.ipme.ru/ipme/labs/iisad/sapr.htm, E_mail: risk@sapr.ipme.ru.

The bribe LP-models and the corresponding Software are intended for the department “Economic crimes” of the town with the purpose of revealing bribes according to the statistical data.

Basic results of the present work are the following:

1. It is offered to use the risk LP-theory with groups of incompatible events for the development of the bribe LP-models with the purpose of revealing, estimating and analyzing their probabilities on the evidence of the statistical data.
2. The construction of the LB- model comprises the following:

⩽ The presentation of the L-model of the bribe in PDNF in order to estimate the number of various possible combinations of bribes and the computing complexity of algorithms;

⩽ If the computing complexity is great, it should record the bribe L-model in DNF, using the scenario of the bribes in the form of remarks, or with help of the graph, or in the form of the shortest ways; or for the limited set of bribes.

⩽ The transformation of the L-model of a bribe from DNF into ODNF;

⩽ Recording the B-model of the bribe according to ODNF;

⩽ Identifying the B-model of the bribe according to the statistical data taking into account of GIE;

⩽ Analysis of the B-model of the bribe with the calculation of the contribution of the signs and grades into the possibility of a bribe, the average probability of bribes and the exactness of the LB-model.

3. Scenarios and the LP-models are described for bribes:

⩽ at the institutions according to the results of their functioning,

⩽ of the officials on the basis of the descriptions of their behavior,

⩽ at the institutions and of the officials on the basis of the analysis of service parameters.

4. Examples are given of the estimation and analysis of the probabilities of bribes on the basis of identifying the bribe LP-model on the statistical data.
5. The developed LP-models of bribes can be used both individually and all together.
6. Software has been developed for training the risk LP-model, and for the estimation and analysis of the probabilities of bribes. It is intended for the department “Economic crimes” of towns with the purpose of revealing bribes on statistical data.

References

Albrecht, W., Wernz, G., Fraud, Williams T. (1995). *Bringing Light to the Dark Side of Business*. Transl. from English. Saint Petersburg: Piter. 396.

Satarov, G. A. (2004). *Anticorruption politics*. The manual, M.: RA SPAS. 368 .

Eliseeva, I. I. (2004). *General statistics theory: The text book*. M.: Finance and statistics. . 656.

Heckman J. J., Leamer, E. (2002). *Handbook of Econometrics*.

Solozhentsev, E. D. (2006). *Scenario Logic and Probabilistic Man Agement of Risk in Business and Engineering*. Second Edition. Saint Petersburg. Business - Press. 560.

Ryabinin, I. A. (2000). *Reliability and Safety of Structure-Complex Systems*. Saint Petersburg: Politecknika. 248.

Solojentsev, E. D. (2004). *Scenario Logic and Probabilistic Management of Risk in Business and Engineering*. Springer. 391.

Solozhentsev, E.D., Stepanova N.V., Karasev V.V. (2005) *Transparency of Methods for Asessment of Credit Risks and Ratings*. Saint Petersburg: Publishing house St. Petersburg University Press. 196.

Solojentsev, E. D., Karassev V. V., Solojentsev V.E. (1999). *Logic and Probabilistic Models of Risk in Banks, Business and Quality*. Saint Petersburg: Nauka. 120.